APPENDIX C

COMPLEX SEQUENCING STRUCTURES

(ADDITIONAL DEFINITIONS FOR SECTION RELATIONS)

The following additional definitions are not necessary in the context of this study. They do, however, give more insight into the possibilities of the specification and analysis methods discussed.

Definition C.1

Section a_i is called an *immediate successor* of section a_j, and a_j is called an *immediate predecessor* of section a_i, *if* the termination of section a_j as such can enable the initiation of section a_i.

Definition C.2

The *set of immediate successors* of section a_j is symbolically notated as "$S^1(a_j)$". The *set of immediate predecessors* of section a_i is symbolically notated as "$P^1(a_i)$".
Set $S^1(a_j)$ is also called the *first generation of successors* of section a_j.

Definition C.3

The *n-th generation of successors* of section a_j, symbolically notated as "$S^n(a_j)$", is recursively defined as follows:

$$(\forall\ i)\ \{a_i \in S^n(a_j) \rightarrow (\exists\ k)\ \{a_k \in S^{n-1}(a_j) \rightarrow a_i \in S^1(a_k)\}\}.$$

Definition C.4

A coordination graph in which for each section a_j the following condition is fulfilled is called an *A-graph*:

$$(\forall\ i)(\forall\ j)(\forall\ h)\ \{a_i \in S^1(a_j) \cap a_h \in S^1(a_j) \cap (h \neq i) \rightarrow a_h \not\equiv a_i \cap P^1(a_i) \equiv P^1(a_h)\}.$$

Note that any coordination graph can be transformed into an equivalent A-graph by introducing new (dummy) sections.
In figure C.1 we have indicated the three relevant types of transformations:

C.1.a: If $a_h \equiv a_i$, we can introduce a new section $a_k \not\equiv a_h$ in place of of a_i

C.1.b: If $P^1(a_i) \not\equiv P^1(a_h)$ we can make $P^1(a_i)$ and $P^1(a_h)$ completely disjoint, so that:

$$(a_i \in S^1(a_j) \cap a_h \in S^1(a_j) = \text{FALSE})\ \text{(see also fig. C.1.c)}.$$

In figure C.2 a complete transformation is shown for an arbitrary coordination graph which is no A-graph initially. The new sections (introduced in the transformation) are shaded.

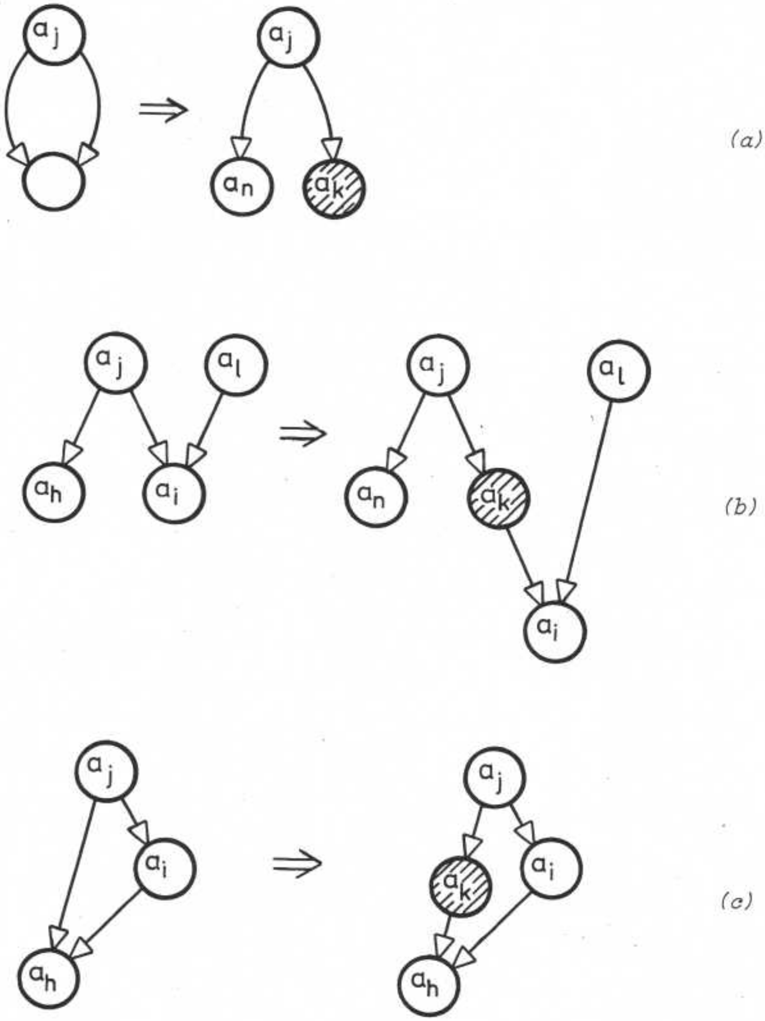

Figure C.1.
Transformations to A-graphs

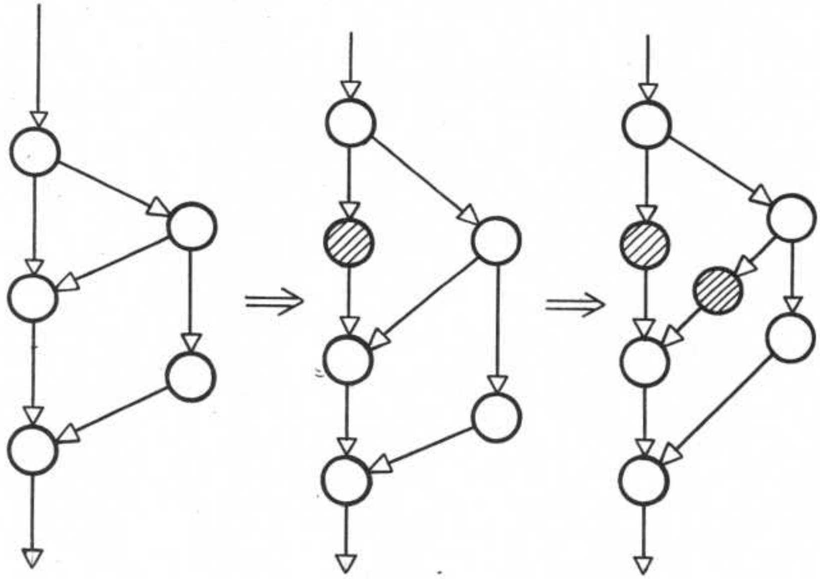

Figure C.2.
Transformation in Two Steps

We state the following property for A-graphs:

In an A-graph:

$$a_i \in S^1(a_j) \cap a_h \in S^1(a_j),$$

implies:

$$P^1(a_i) \equiv P^1(a_h),$$

and:

$$a_j \in P^1(a_i) \cap a_k \in P^1(a_i),$$

implies:

$$S^1(a_j) \equiv S^1(a_k).$$

Proof:

Suppose that $S^1(a_j) \not\equiv S^1(a_k)$.
The implication is then that:

$$\textit{Either:} \quad (\exists\, m)\ \{a_m \in S^1(a_j) \rightarrow a_m \notin S^1(a_k) \rightarrow a_k \notin P^1(a_m)\}. \qquad (1)$$

The assumption is, however, that $a_j \in P^1(a_i)$ or $a_i \in S^1(a_j)$, which (by definition) means:

$$P^1(a_i) \equiv P^1(a_m). \qquad (2)$$

As we also assumed that:

$$a_k \in P^1(a_i), \qquad (3)$$

we may conclude (from (2) and (3)) that $a_k \in P^1(a_m)$. This contradicts clearly with (1).

Or: $(\exists m)\ \{a_m \in S^1(a_k) \rightarrow a_m \notin S^1(a_j) \rightarrow a_j \notin P^1(a_m)\}.$ (4)

The assumption is again, that $a_i \in S^1(a_j)$ or $a_j \in P^1(a_i)$, which (by definition) means that:

$$P^1(a_i) \equiv P^1(a_m). \tag{5}$$

We also assumed that:

$$a_i \in S^1(a_j). \tag{6}$$

so that we can conclude from (5) and (6) that $a_j \in P^1(a_m)$. This also contradicts with (4).
This completes the proof.

We have proved that all successors of a specific section in an A-graph have the same predecessors, and vice versa. Instead of speaking of a predecessor-successor relation, we can therefore in A-graphs speak of relations between *predecessor-sets* and *successor-sets*.
Below we discuss 4 practical section-set relation types.

(a) We can model the situation in which for *each* termination of a section from a predecessor-set, precisely *one* section from the corresponding successor-set is enabled to initiate once, as follows (see figure C.3), by making:

$$(\forall j)\ \left\{(1 \leq j \leq N^S_\alpha) \rightarrow \left[\sum_{i=1}^{N^P_\beta} T_{p_i(\beta)} - \sum_{k=1}^{N^S_\alpha} I_{s_k(\alpha)} > \emptyset\right] = sic_{s_j(\alpha)}\right\}.$$

In which the following symbols are used:

$\alpha = P^1(\beta)$

$\beta = S^1(\alpha)$

$p_i(\beta)$ - denotes the i-th immediate predecessor of set β;

$s_j(\alpha)$ - denotes the j-th immediate successor of set α;

N^S_α - denotes the number of sections in set β (successors of α);

N^P_β - denotes the number of sections in set α (predecessors of β).

(b) We can model the situation in which for *each* termination of a section from a predecessor set, *all* sections from the corresponding successor set are enabled to initiate once, as follows (see figure C.4), by making:

$$(\forall j)\ \left\{(1 \leq j \leq N^S_\alpha) \rightarrow \left[\sum_{i=1}^{N^P_\beta} T_{p_i(\beta)} - I_{s_j(\alpha)} > \emptyset\right] = sic_{s_j(\alpha)}\right\}.$$

Set α can be called the *split-set* of set β, and
set β can be called the *parallel successor-set* of set α.

(c) One initiation of *one* section from a successor-set is enabled if *all* sections from the corresponding predecessor-set have terminated once, with the following rules (see also figure C.5):

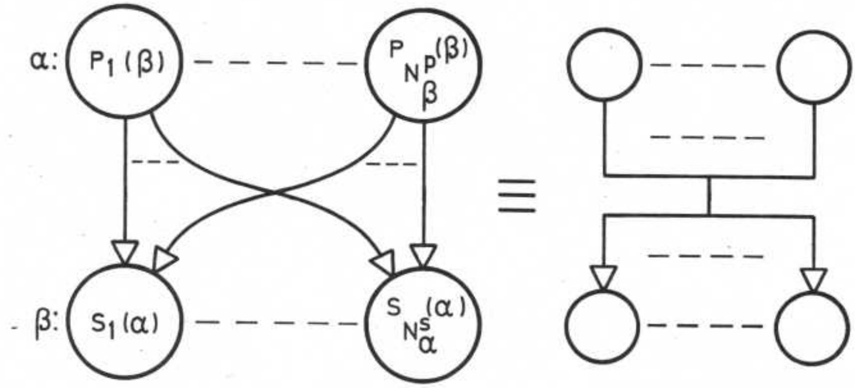

Figure C.3.
Simple Succession

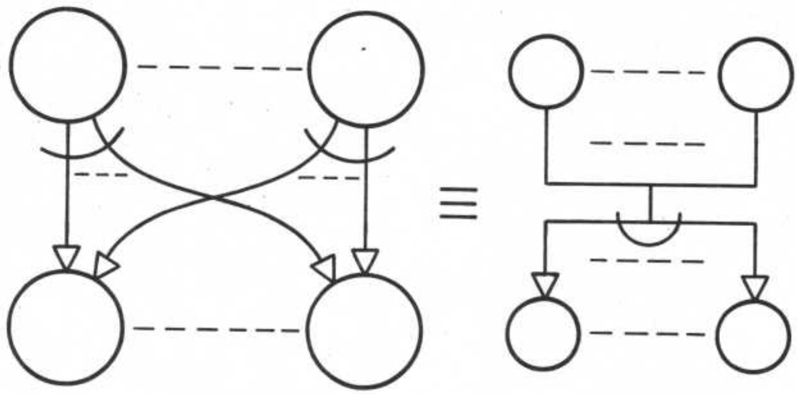

Figure C.4.
Split Succession

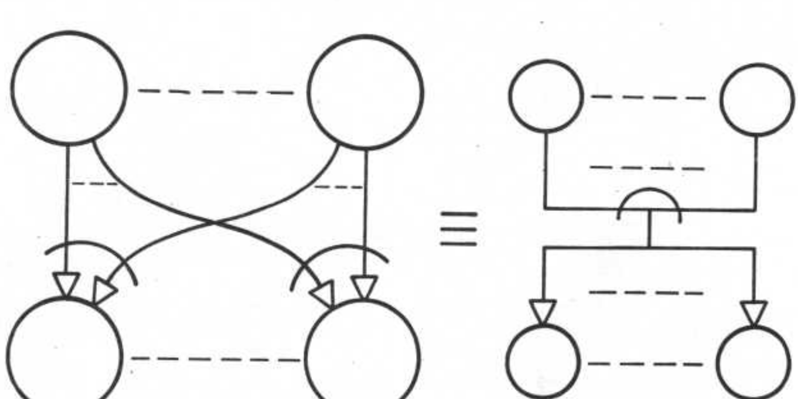

Figure C.5.
Join Succession

$$(\forall\, j)\ \left\{ (1 \leqslant j \leqslant N^{s}_{\alpha}) \rightarrow \left[\mathop{\Delta}_{i=1}^{N^{p}_{\beta}} T_{p_i(\beta)} - \sum_{k=1}^{N^{s}_{\alpha}} I_{s_k(\alpha)} > \emptyset \right] = sic_{s_j(\alpha)} \right\}.$$

Set α can be called the *parallel predecessor-set* of set β, and set β can be called the *join-set* of set α.

(d) One initiation of *all* sections from a successor-set are enabled if *all* sections from the corresponding predecessor-set have terminated once, with the following rules (combination of b and c):

$$(\forall\, j)\ \left\{ (1 \leqslant j \leqslant N^{s}_{\alpha}) \rightarrow \left[\mathop{\Delta}_{i=1}^{N^{p}_{\beta}} T_{p_i(\beta)} - I_{s_j(\alpha)} > \emptyset \right] = sic_{s_j(\alpha)} \right\}.$$

Set α can be called the *parallel predecessor-set* of set β, and set β can be called the *parallel successor-set* of set α.

Further extensions to the definitions are of course still possible, but perhaps less practical. For instance, one can introduce fractional *weight factors* for each relation between two (sets of) sections, which indicate *how many times* each successor can initiate for each single termination of the predecessor(s). Above we have assumed weight factors equal to 1.